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Invited Comment

Weak-value measurements can outperform conventional measurements

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Abstract

In this paper we provide a simple, straightforward example of a specific situation in which weak-value amplification (WVA) clearly outperforms conventional measurement in determining the angular orientation of an optical component. We also offer a perspective reconciling the views of some theorists, who claim WVA to be inherently sub-optimal for parameter estimation, with the perspective of the many experimentalists and theorists who have used the procedure to successfully access otherwise elusive phenomena.

Keywords: quantum measurement, weak values, quantum wavefunction

(Some figures may appear in colour only in the online journal)

1. Introduction

A lively debate has recently been sparked in the scientific literature, regarding the question of whether weak-value measurements (WVMs) [1, 2] can outperform conventional measurements in determining a physical parameter of a system. One line of reasoning concludes that WVMs cannot be used to increase measurement precision, because, stated most simply, WVMs entail the use of post-selection. Data is discarded in the post-selection process, and from an information-theoretical standpoint, one is always worse off for having rejected data. This argument can be formulated very rigorously, and is quite compelling. Even if the data that are retained are vastly more 'information-rich' per datum, there is undoubtedly *some* useful information in the discarded data, and one could make a more accurate measurement by retaining this information.

However, there are potential flaws in this argument. First of all, while post-selection and sorting can result in a loss of available information [3], weak value-based measurements do not necessarily require that data be discarded. Instead, data can be sorted into bins by the means of post-selection, and each bin subsequently used for a specific measurement. This strategy has been successfully implemented in the past [4, 5].

Another potential flaw is that, even in a conventional measurement, there may be practical factors that preclude the use of all of the data that are in principle available in theory. In such instances, the experimentalist is faced with the choice of either discarding data indiscriminately, or discarding only the least information-rich data. Experimentally, the latter option would clearly result in a stronger signal than the former, conferring greater accuracy upon measurements of the quantity of interest. This, in a sense, is the entire point of weak value amplification (WVA). One might choose to discard the vast majority of the data, which contains little useful information, and instead retain that fraction of the data for which the signal to be measured is large [6]. The presence of certain types of technical noise in the detection system represents one example of a situation in which this strategy may be desirable, as WVA could ensure that the signal to be measured lies above the noise floor of the detection system.

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Here, we provide an overview of the WVA versus conventional measurement controversy, and present a simple example of a situation in which WVA affords an advantage over strong measurement.

2. Background

Conventional (or 'strong') measurement in quantum metrology can be implemented or modelled by an interaction between two quantum systems. A fairly typical example in optics is the interaction between the polarization and position degrees of freedom of a beam of light, though other approaches are also possible (for example, coupling between a beam's orbital angular momentum (OAM) and time or frequency). For clarity, we will refer to these two interacting degrees of freedom as the 'system' and 'pointer', and will denote the corresponding quantum mechanical observables by $\hat{A}$ (the system) and $\hat{x}$ (the pointer). If the respective initial states of the system and pointer are denoted by $|I\rangle$ and $|\psi\rangle$, then the interaction results in a transformation $|I\rangle|\psi\rangle \to \mathrm{e}^{-\mathrm{i}g\hat{A}\hat{p}_x/\hbar}\,|I\rangle|\psi\rangle$ in the overall state of the beam [1]. Here, $\hat{p}_x$ is taken to represent an observable Fourier conjugate to $\hat{x}$, and g is a parameter that controls the strength of the interaction between these two degrees of freedom. For concreteness, $\hat{x}$ and $\hat{p}_x$ might respectively be imagined to represent the x-position and linear momentum operators associated with the transverse degree of freedom of a light beam. At the same time, $\hat{A}$ could be a polarization observable, taken to have eigenvalues $a_\pm = \pm 1$, associated with its horizontal (+) and vertical (−) polarization eigenstates $|H\rangle$ and $|V\rangle$, respectively. For the simple case in which a beam is prepared in an initial state $|I\rangle = |H\rangle$, conventional measurement involves a transformation of the form $|H\rangle|\psi\rangle \to \mathrm{e}^{-\mathrm{i}g\hat{A}\hat{p}_x/\hbar}\,|H\rangle|\psi\rangle = \mathrm{e}^{-\mathrm{i}g\hat{p}_x/\hbar}\,|H\rangle|\psi\rangle$. The term $\mathrm{e}^{-\mathrm{i}g\hat{p}_x/\hbar}$ serves as a translation operator, so that the final state thus receives a 'kick' in position space by an amount $a_m g$, with $m = +$ when $|I\rangle = |H\rangle$ and $m = -$ when $|I\rangle = |V\rangle$. As a result, the final state in the position basis will be described by a wavefunction $\langle x|\,\psi\rangle = \psi(x - a_m g)$, which now carries information about the interaction parameter g. Indeed, g can be estimated by simply measuring the position of the beam's centre before and after the position/momentum interaction. Consequently, while this interaction was introduced as a model of the measurement of system observable $\hat{A}$, this procedure can also be seen as a practical method to measure the interaction strength g itself. In other words, we are seeking to estimate the magnitude of an interaction induced by a physical effect under study. In our above example, it could be the transverse shift induced by a birefringent crystal (e.g. see [2]). Indeed, in this case, g can be estimated by simply measuring the position of the beam's centre before and after the interaction.

It has been suggested that WVA might allow for improved measurements of g relative to the more conventional scenario just presented. In order to implement the WVA scheme, one must prepare the beam's initial state in a superposition of the eigenstates of $\hat{A}$, for example, $|I\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$. Next, the interaction above occurs, in the manner already described for conventional measurement. Thus, the beam takes the form $|I\rangle|\psi\rangle \to \mathrm{e}^{-\mathrm{i}g\hat{A}\hat{p}_x/\hbar}\,|I\rangle|\psi\rangle$. Note that $|I\rangle$ is not a polarization eigenstate of $\hat{A}$. We now suppose that a post-selection is carried out, in which the beam is made to pass through a polarization filter, which guarantees that the polarization state of the emerging beam is given by $|F\rangle$. Assuming g to be small, the position state of the beam will then be given by [1]

$$\begin{aligned} |\psi_{\mathrm{WVA}}\rangle &= \langle F|\mathrm{e}^{-\mathrm{i}g\hat{A}\hat{p}_x/\hbar}|I\rangle|\psi\rangle \\ &\approx \langle F|(1 - \mathrm{i}g\hat{A}\hat{p}_x/\hbar)|I\rangle|\psi\rangle \\ &\approx \langle F|I\rangle(1 - \mathrm{i}gA_{\mathrm{w}}\hat{p}_x/\hbar)\,|\psi\rangle \\ &\approx \langle F|I\rangle\,\mathrm{e}^{-\mathrm{i}gA_{\mathrm{w}}\hat{p}_x/\hbar}|\psi\rangle, \end{aligned} \tag{1}$$

where we have neglected terms $O(g^2)$ and above, and have introduced $A_{\mathrm{w}} \equiv \langle F|\,\hat{A}\,|I\rangle/\langle F|\,I\rangle$, the weak value of the observable $\hat{A}$, which we take to be real for the moment, by making an appropriate choice of $|I\rangle$ and $|F\rangle$. When normalized, the WVA wavefunction thus takes the form $\langle x|\,\psi_{\mathrm{WVA}}\rangle/\langle F|\,I\rangle = \psi(x - \mathrm{A_w}g)$. A_{w} can, in general, take on values that lie well outside the eigenspectrum of $\hat{A}$, so that $A_{\mathrm{w}} > \max(A)$, particularly when $\langle F|\,I\rangle \approx 0$. As a result, when g is small, the WVA approach can produce shifts in the beam's system degree of freedom far exceeding those produced in the case of conventional measurement (1). This is true in the 'weak measurement regime' defined by $A_{\mathrm{w}}g/\sigma \ll 1$, where σ is the standard deviation of the pointer position (i.e. the transverse beam width) [7]. Outside this regime, the expansion in equation (1) is not valid and the pointer position distribution (i.e. the beam shape) will be distorted by the interaction.

Based on equation (1) alone, the WVA strategy certainly seems promising; the expression suggests that otherwise minute or imperceptible signals can be enlarged by increasing the weak value A_{w}, presumably resulting in improved sensitivity and heightened signal-to-noise ratios (SNRs) on estimations of g. However, this conclusion overlooks an important consideration: in reality, the desirable 'amplification' effect is generally offset by a corresponding decrease in the strength of the post-selected signal obtained following the weak interaction [8]. In effect, amplification can be achieved only at the cost of 'throwing away' data, a process that is in general associated with a loss of available information. Specifically, only the fraction $|\langle F|\,I\rangle|^2$ of photons remain after the post-selection filter. In weighing the merits of the WVA approach, it then becomes necessary to account for both the signal amplification due to the (potentially) large weak value, and the signal loss necessarily incurred as a byproduct of post-selection.

Considerable effort has recently been expended in attempts to determine whether WVA actually confers an overall advantage in quantum metrological applications, once these (and other) competing effects are accounted for. Unfortunately, this work has produced a wide range of results which may at first glance appear to be mutually inconsistent, or even contradictory. One could be forgiven for experiencing some confusion when confronted with theoretical studies that purport to show the WVA approach to be undesirable [9, 10], given that several experimental investigations have successfully applied the WVA strategy to the measurement and detection of otherwise inaccessible phenomena. Notably, WVA was used to measure photon displacements to within $\sim$1 Å, in a demonstration of a photonic analogue to the quantum spin Hall effect [11], and has also been successfully applied to the ultrasensitive measurement of the displacement of optical components [12], and beam rotations [13].

Despite this debate, it is generally accepted that in the complete absence of noise, detector saturation and other practical constraints (i.e. in the experimentally unattainable limit of a 'perfect measurement'), the WVA technique will not confer an advantage over standard (strong) measurement schemes. For this reason, those advocating for WVA's metrological potential generally argue that the benefits of the post-selection approach are felt under explicitly sub-optimal experimental conditions. Indeed, the recent controversy surrounding the usefulness of WVA in quantum metrology was ostensibly sparked by an argument made by Feizpour, Xing and Steinberg (FXS), regarding the potential benefits afforded by WVA in the presence of highly correlated noise [14]. In their paper, FXS use a weak-measurement strategy to amplify nonlinear phase shifts associated with single photons, and estimate these phase shifts by averaging their data. Their suggestion that WVA might prove superior to standard measurement was countered by Ferrie and Combes (FC), who argued that the superior SNRs reported by FXS using the weak measurement scheme were artifacts of the team's use of a sub-optimal estimation strategy [15]. Rather than a simple average over one's data, FC pointed out that the rigorously correct estimator (that associated with the lowest mean standard error) is always the maximum likelihood estimator (MLE). FC's theoretical treatment revealed that when the MLE is employed rather than the mean, not only does the advantage of the FXS WVA strategy disappear, but the approach results in a *decrease* in SNR. However, the MLE requires knowledge of a covariance matrix K, which defines the amplitude of the noise in the quantity being measured. As one rarely has access to this information, it might be argued that the MLE cannot be implemented in practice. FC anticipate this objection by demonstrating that a simplified MLE can be defined, which does not depend on this covariance matrix, and which is unbiased and nearly optimal when $\sigma^2 \gg \|K\|$. FC also point out that FXS's use of SNR as a figure of merit in evaluating WVA is, itself, undesirable, as it does not reflect the performance of the MLE, but rather that of a sub-optimal estimator, and argued that the mean standard error should be used instead, concluding from their analysis that, '… there is no sense in which WVA provides an 'amplification' for quantum metrology.'

FC's theoretical exposition was followed in short order by a response from Vaidman [16], who argued that experimental conditions are generally such that detector saturation can play a significant role in constraining measurements, and implied that once this effect was taken into consideration, WVA would indeed allow for improved measurement precision. Jordan, Martínez-Rincón and Howell (JMH) shortly thereafter reported a detailed theoretical treatment of the purported benefits of the WVA approach (and specifically considered the role played by imaginary weak values) in the presence of a number of specific types of noise, and argued that the strategy might be used to remove the effects of detector noise and air turbulence in beam deflection measurements [9]. JMH investigated the case of arbitrary metre wavefunctions and technical noise that is not time-correlated, and concluded that although no benefit can be derived from amplification using the real part of the weak value (in agreement with FXS), a WVA technique implementing an imaginary weak value *can* afford an advantage. JMH also made the important point that benchmarking the performance of WVA using rigorously optimal estimators, as in the strategy adopted by FC, may not always be appropriate. 'Rather,' they argued, 'the optimal estimator should be found and must be practically implementable. If it is not, the inefficient—but practical—estimator is advantageous.' Returning to the case made by FXS, JMH suggest that the case of time-correlated technical noise is precisely one in which the optimal estimation strategy is not practically feasible, respectively making the SNR and experimental averaging the appropriate figure of merit for measurement performance and parameter estimator at least in practice, if not in principle. That WVA improves on standard measurement when the SNR is used as the measure of performance was also demonstrated by Pang and Brun, for measurement pointers prepared in squeezed coherent states [17], though it remains somewhat unclear whether this advantage is due to WVA itself, or merely to the SNR improvement that generally accompanies squeezing.

Criticism of the WVA technique is often expressed in the more robust language of Fisher information (FI), a quantity that effectively provides the variance of the best possible estimator for a parameter of interest [18]. Different experiments carried out on the same system can provide different FIs: one might readily imagine, for example, an experimental configuration that is so sub-optimal that essentially no information is provided about the parameter being interrogated (one might say that such an experiment was 'poorly designed'), or conversely, an experiment so well designed that it extracts all the information about this parameter theoretically contained in the quantum state. The FI obtained in the latter case is referred to as the quantum Fisher information (QFI), and is strictly defined as the FI, maximized over the set of all measurements (POVMs) that might theoretically be carried out on the system [19]. To this extent, the QFI effectively becomes a property of the state being probed, rather than being characteristic of the measurement process itself.

Tanaka and Yamamoto demonstrated that the full QFI contained in a quantum system about a given parameter cannot in general be extracted by the WVA approach, under asymptotic conditions (i.e. for an infinite number of photons) [20]. Knee and Gauger came to a similar conclusion by making use of a model that traced out (and therefore ignored) the system, making use of no post-selection [10]. These arguments were later extended by Combes, Ferrie, Jiang and

Caves (CFJC), who framed their position in the language of density matrices, allowing them to draw conclusions applicable in the case of finite photon counts and, in principle, to generalize their results to include the effects of noise [21]. In their paper, CFJC explicitly indicate at which stages and by what mechanisms QFI can be lost during a typical WVA measurement. To this end, they divide a typical WVA measurement into four stages. In the first, a joint state is prepared, consisting of a distinct system, which contains information about some parameter x of interest, and ancilla degrees of freedom. Next, the system and ancilla are made to interact, resulting in the storage of information in the ancilla. The newly entangled system-ancilla is then partitioned into a category of information-rich outcomes, and one of unfavourable outcomes. The latter are discarded in the final stage, resulting in a post-selected system state into which information about x has been concentrated. At no stage, CFJC argue, can the QFI about x have increased. Further, as information is necessarily discarded in the third and final stages, the QFI can only decrease or, at best remain constant, in a WVA measurement. CFJC condition their conclusion that WVA can only lead to a loss of available information on two premises: (1) 'a sub-optimal strategy cannot achieve optimal performance,' and (2) 'information cannot be increased by throwing some of it away.'

A related argument was developed by Zhang, Datta and Walmsley (ZDW), who considered the partitioning of FI in post-selected weak measurement schemes [3]. ZDW showed that a typical WVA experiment distributes information about the parameter(s) of interest in three different places: the success modes (the modes that survive post-selection), the failure modes, and the distribution of the post-selection process itself. That is, they consider the case in which all data are retained. The total FI after a WVA-based measurement is then given by

$$F_{\text{tot}} = p_d \mathcal{F}_d + (1 - p_d)\, \mathcal{F}_r + F_p,$$

where $\mathcal{F}_d$, $\mathcal{F}_r$ and F_p respectively denote the QFI in the success mode, the QFI in the failure mode, and the (classical) FI associated with the distribution of the post-selection process $\{p_d, 1 - p_d\}$. In a typical WVA experiment, one discards the failure modes, leading to a loss of available information $(1 - p_d)\mathcal{F}_r$. Since the total FI after the weak interaction between system and pointer cannot be greater than the QFI, $\mathcal{F}_i$, present in the original state (prior to interaction and post-selection), ZDW argue that $\mathcal{F}_i \geqslant F_{\text{tot}} \geqslant p_d \mathcal{F}_d + F_p$. They observe that while $\mathcal{F}_i < \mathcal{F}_d$ is possible, the low post-selection probability p_d ensures that overall, WVA cannot result in a net gain of information.

The existence of a substantial body of work demonstrating the sub-optimality of WVA under ideal conditions makes all the more surprising the findings of Bié Alves *et al* who recently provided a prescription for state post-selection in WVA-type schemes, which allows the full QFI $\mathcal{F}_i$ originally contained within a quantum state to be extracted, up to first order in the coupling parameter g [22]. They show that in certain measurement regimes the original QFI is concentrated almost entirely in the distribution of the post-selection process, or in the successfully post-selected mode. According to this line of argument, very little is sacrificed by making use of WVA rather than a conventional strong measurement strategy, even under optimal circumstances. In addition to demonstrating WVA's potential to realize an optimal or near-optimal measurement, Alves *et al* derive the post-selected states for which the FI approaches the QFI, on the assumption that the meter used for the measurement is balanced. Their analysis leads to the surprising conclusion that exact orthogonality between pre- and post-selected states, , which is generally avoided in WVA experiments, can actually maximize the FI obtained in a WVA measurement. This is because the case $\langle I \mid F \rangle = 0$ can result in a concentration of FI in the statistics of the post-selection distribution, rather than in the meter.

Others have reached similar conclusions, regarding the ability of WVA to saturate the QFI originally available in the quantum system prior to post-selection [9]. Alternative strategies have also been proposed to further improve the extent to which this upper limit can be saturated by WVA, based, for example, on entangling additional ancillary states with the system state [23].

Ultimately, the current controversy surrounding the possible benefits of WVA is perhaps no more than a byproduct of binary thinking, rather than any actual disagreement within the literature. Some criticize WVA, owing to its failure to outperform conventional measurement under ideal or near-ideal circumstances. A second group argues that experimental context matters, pointing out that WVA plays an important practical role in real-world measurement, due to the existence of such constraints as noise and detector saturation. These two positions are not incompatible; taken together, they merely suggest that there exists a range of experimental conditions under which conventional measurement should be preferred to WVA, and one under which this state of affairs is reversed. A number of experimental observations certainly suggest that, in practice, the frequency with which the experimentalist encounters the latter scenario is far from negligible [6, 11–13].

3. Example: a practical advantage for weak value amplification

We now provide a simple example of a situation in which WVA allows us to measure an otherwise inaccessible physical parameter. We consider the situation shown in figure 1. Part (a) of the figure shows the numerically simulated intensity profile of light beam that has the form of an angular wedge. More precisely, the beam has a transverse field profile that is Gaussian in the azimuthal coordinate, $\psi(\phi) = \mathrm{e}^{-\phi^2/\eta^2}$ where η denotes the intensity $1/e^2$, half-width of the wedge, which is taken to be 28.6° in this example. The wedge is centred on the horizontal axis. The challenge is to determine, by eye alone, whether the wedge has been rotated. Part (b) of the figure shows the same wedge, rotated counterclockwise by 0.5°. Even under close inspection, the rotation of the

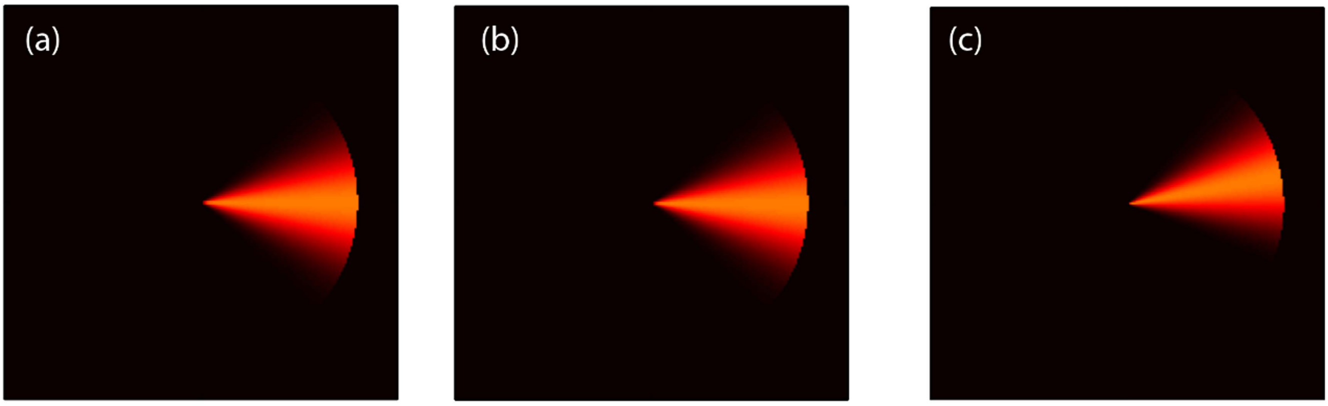

Figure 1. A simple simulated example of the utility of weak value amplification. (a) A light beam with a Gaussian azimuthal intensity transverse spatial profile, i.e. a wedge. (b) The wedge of part (a) has been rotated by 0.5°. There is no noticeable change in the figure. (c) After weak value amplification, the rotation of the wedge is increased by a factor of 22.5, and the rotation is clearly visible by eye. The intensity in (c) has been increased by a factor of roughly 1000 so that it is visible in the plot.

angular wedge eludes the naked eye. One can perform a weak value amplification of the rotation angle by using the method described by Magaña-Loaiza *et al* [13]. The simulated resulting intensity profile is given in Part (c). As discussed below, even by eye, it is clear that the structure has been rotated and has been rotated in a counterclockwise sense.

One may be tempted to argue that this comparison is unfair, as it overlooks the beam intensity loss due to post-selection; indeed, in this situation, the image of part (c) would be about 1000 times weaker than that of parts (a) or (b). But this fact is irrelevant for the point that we are making here. The human visual system possesses a dynamic range that is at least this large [24, 25]. Attenuation of a laser signal by a factor of 1000 still allows the structure of the laser beam to be discerned. Or stated differently, amplifying a laser beam by a factor of 1000 does not usually make it easier for a human observer to discern the structure of the beam. Hence, the experimental context—in this case, the fallible and dynamic human eye—makes a great difference in determining whether and to what extent the WVA approach is viable.

The example given here assumed that the measurement was to be made by the human visual system. Nonetheless, many of the same considerations would apply to state-of-the-art photodetection systems. The point here is that, when using certain realistic detection strategies, it is simply easier to determine from figure 1(c) than from figure 1(b) that beam has been rotated away from the horizontal symmetry plane. We believe that such practical considerations cannot be overlooked when weighing the merits of WVA. In the next section, we go into detail about an experimental setup that can produce this striking example.

4. Numerical simulations of the weak value amplification of angular rotations

The study of weak measurements and, by extension, of weak value amplification originated in the field of quantum measurement theory. WVA has been recognized as an interference phenomenon [26–28]. Indeed, the various WVA protocols that have been proposed since the technique's inception have been implemented in optical interferometers. Here, we take advantage of the interference-based origin of WVA, and invoke interference arguments to illustrate a situation in which weak measurement is advantageous relative to a conventional measurement. As a specific example, we discuss a protocol for measuring the angular rotation of an optical element. Similarly to the first demonstration of WVA performed by Ritchie and colleagues [2], this protocol exploits the spatial degree of freedom of light, which is independent of the excitation mode of the field. Consequently, our experiment can be explained using classical arguments.

The first protocol for WVA in the azimuthal variables of angular position and OAM demonstrated the possibility of amplifying small angular rotations [13]. This protocol was realized experimentally using a Sagnac interferometer in which was placed a Dove prism. It was demonstrated that the real weak value produces a shift in the angular position of an optical beam, whereas the imaginary weak value induces a shift in the OAM spectrum of the beam. The use of real weak values for amplification of angular rotations provides a direct and simple means of visualizing the practical advantage of weak measurements and post-selection. By contrast, the physics of imaginary weak values in the azimuthal degree of freedom must be analysed using more sophisticated tools, such as geometric phases and interference in the OAM spectrum, which serve to obscure the origin of the amplification effect. We shall therefore set aside the high amplifications that have been achieved by using imaginary weak values, focusing our attention instead on the consequences of real weak values. A formal discussion of imaginary weak values in the context of the azimuthal degree of freedom can be found in reference [13].

As discussed in section 3, we use a beam with a transverse profile of the form of a Gaussian wedge. This can be generated using a spatial light modulator. As shown in figure 2, this angular mode is prepared in a diagonal polarization state using a polarizer and a half wave plate. We assume that this beam is injected into one of the input ports of

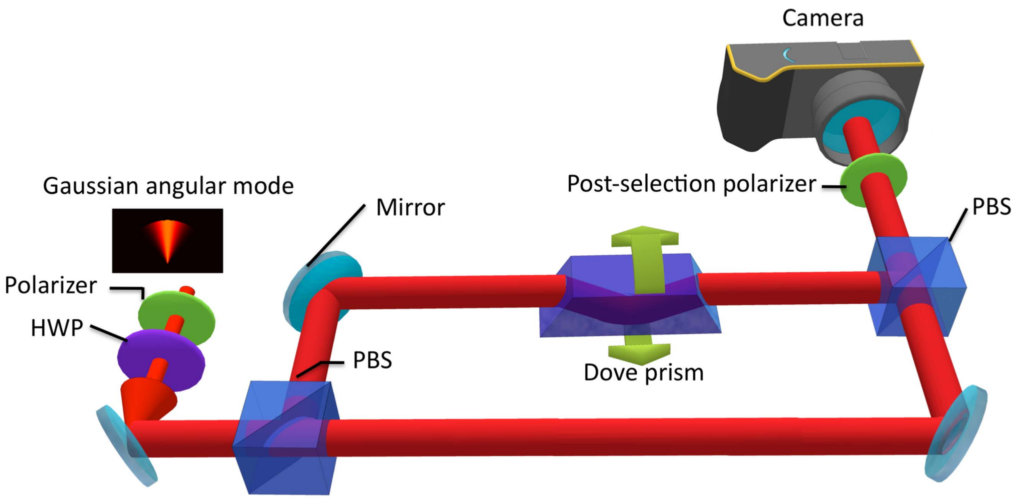

Figure 2. Experimental setup. An angular wedge is prepared in diagonal polarization by means of a polarizer and half wave plate (HWP). The beam is split by a polarizing beam splitter (PBS) and rotated by a Dove prism, the beam is rotated by twice the angle of the Dove prism. A post-selection polarizer is used to amplify the rotation induced by the Dove prism.

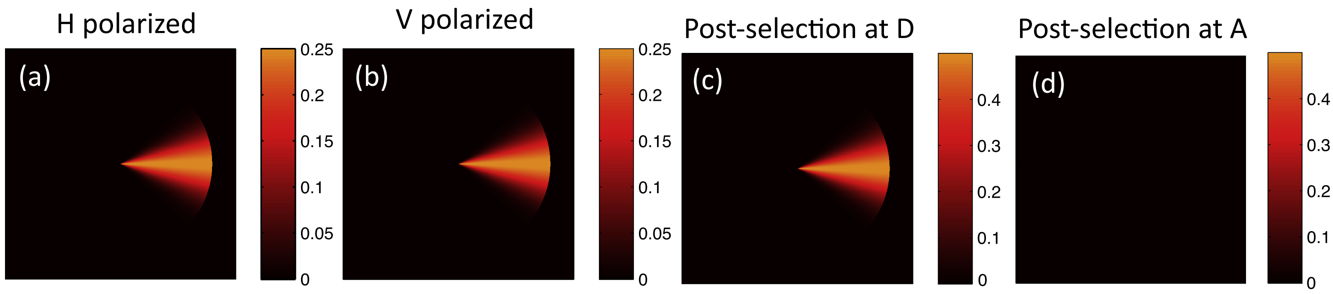

Figure 3. Simulation results. (a) Shows the spatial profile of the Gaussian wedge in one of the arms of the interferometer. The polarization of this beam is aligned in the horizontal direction. (b) Shows the vertically polarized beam in the second arm. (c) Shows constructive interference when the position of the post-selection polarizer set diagonally. (d) Shows destructive interference when the post-selection polarizer is anti-diagonal.

a polarization-sensitive Mach–Zehnder interferometer. A polarizing beam splitter (PBS) divides the beam into its horizontally and vertically polarized components. The vertically polarized beam passes through a Dove prism, and the two beams are then recombined in a second PBS. The parameter we seek to measure is the rotation angle of the Dove prism. The Dove prism will rotate the vertically polarized beam by twice its angle. The interaction strength g is the prism rotation angle. A final polarizer comprises the post-selection. The beam's transverse intensity profile is then measured by a camera. This simple experimental setup has been applied to other sophisticated experiments that have been used to test counterintuitive effects in quantum mechanics [29].

We now proceed to consider the conventional and WVA measurement techniques in turn, beginning with the latter, which is found to outperform its alternatives in most experiments involving classical interferometry. We consider the case in which a Gaussian wedge-shaped beam with $\eta = 28.9°$ is injected into the input port of the setup depicted in figure 2. For the moment, we assume that the Dove prism is not rotated. Figures 3(a) and (b) show the spatial profile of this angular wedge. In this case, the output port of the PBS is expected to be brightest when the angle of the Dove prism is zero and the angle of the post-selection polarizer is set to diagonal; this result is illustrated in figure 3(c). When the post-selection polarizer is set to anti-diagonal, however, the two angular modes interfere destructively and the port becomes dark, see figure 3(d). Given that we begin with a diagonal polarization, this shows that the interferometer does nothing when the prism is not rotated, as expected.

We now consider the effect of rotating the Dove prism. We consider the situation in which the Dove prism induces an angular separation of 0.5° between the wedge modes in each arm of the interferometer. As shown in figures 4(a) and (b) this rotation is very small, and can hardly be discerned by eye. In conventional interferometry, this situation might be said to arise as a result of a slight misalignment of the interferometer. The post-selection polarizer is set to anti-diagonal. This rotation of the prism gives rise to interference fringes at the output port of the interferometer, as shown in figure 4(c). Since we are post-selecting on a polarization perfectly orthogonal to the initial polarization it is impossible to satisfy the conditions needed for a weak measurement. In particular, notice the post-selection probability $|\langle F | I \rangle|^2$ is zero in this case. This means that $A_{\mathrm{w}} = \infty$ and, thus, the scheme is

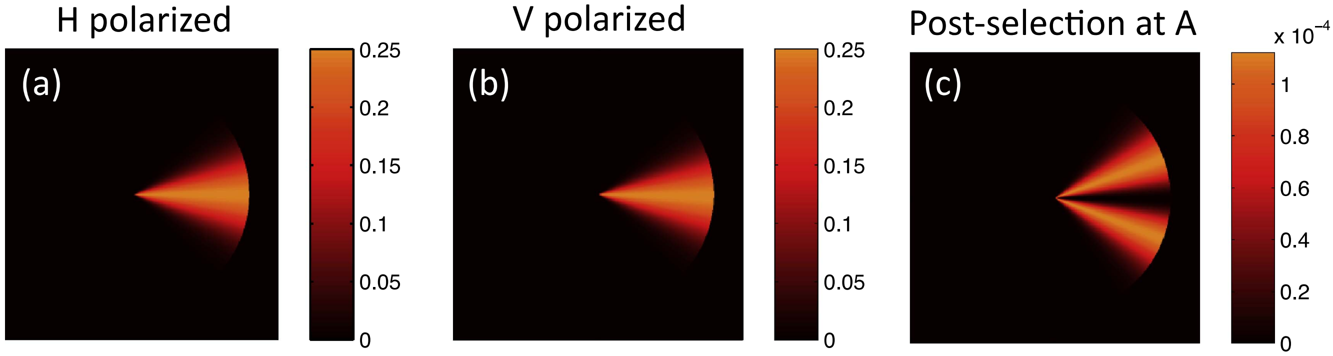

Figure 4. Simulations of a small rotation. The horizontally polarized beam (reference) in one of the arms of the interferomer is shown in (a). The vertically polarized beam is rotated by 0.5°, this is shown in (b). (c) The intensity after an anti-diagonal post-selection polarizer.

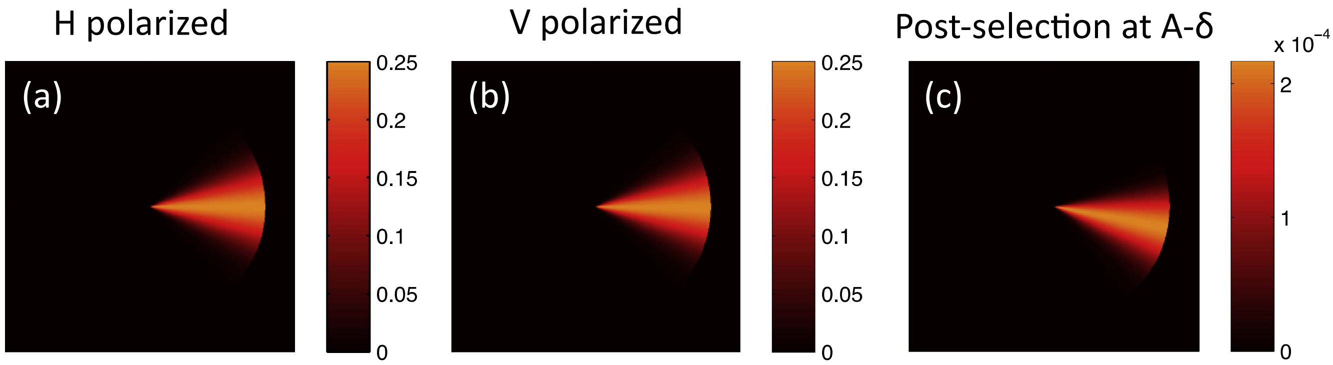

Figure 5. Simulation for the weak measurement regime. The horizontally polarized beam (reference) in one of the arms of the interferomer is shown in (a). The horizontally polarized beam is rotated by 0.5°, as is shown in (b). The rotation has been amplified by a factor of 22, as is shown in (c).

outside the regime in which weak measurement is valid. Consequently, the pointer (our wedge) becomes distorted; it has two lobes.

In the simulations discussed above, the polarizer forces the two beams to interfere constructively or destructively. Interestingly, the polarizer can also be used as a specific filter to discard photons that carry little information about the rotation. We illustrate this approach with the following example: we assume that the wedge mode with horizontal polarization is not rotated, whereas the mode in the arm that is vertically polarized is rotated by 0.5°. As can be seen in the simulations shown in figures 5(a) and (b), it is hard to observe this rotation by eye. Later, the two beams are combined and forced to interfere using the post-selection polarizer. The post-selection process destroys polarization information in the probe, and induces a shift in the pointer, or spatial profile of the beam. The shift is proportional to the product of the small rotation in one of the beams, and the real weak value that is determined by the post-selection polarizer. When the post-selection polarizer is set to an anti-diagonal position minus a small angle $\delta = 1°$, we observe the result in figure 5(c). After post-selection, the angle of the pointer is $-11.2355°$, which corresponds to an amplification of 22.4710. In this case, the maximum intensity is normalized to one for diagonal polarization. Of course, since we are post-selecting on an almost orthogonal state, in anti-diagonal polarization case (figure 5(c)) the intensity is reduced by four orders of magnitude.

It has been argued that protocols for WVA offer nothing more than classical interferometry. We now turn to a practical example that shows that it is easier to observe a small angular rotation when WVA is performed. We begin by considering a situation in which the post-selection polarizer is removed. In this case, the incoherent superposition of both beams leads to a distribution broadened and shifted by the Dove prism's angular rotation. Neglecting this small broadening, the reduced shift thus halves the measurement sensitivity. On the other hand, if the polarizer forces the two beams to interfere in the same manner as in the examples given above (classical interferometry), the resulting interference fringes can be analyzed to infer the position of the prism. For example, figure 6(a) shows the interference pattern obtained by means of conventional interferometry for a situation in which the Dove prism induces an angle between the two interfering beams of 0.5°. Figure 6(b) shows a similar pattern for a situation in which the angular separation between the beams is 1°. It is evident that it is hard to distinguish any difference between the two interference patterns. One would have to perform a careful analysis of the interference fringes in order to determine the value of the small rotation. Consequently, the precision for this measurement is expected to be low. Interestingly, the protocol for WVA leads to clear wedge rotations. In this case it is easier to distinguish the situation in

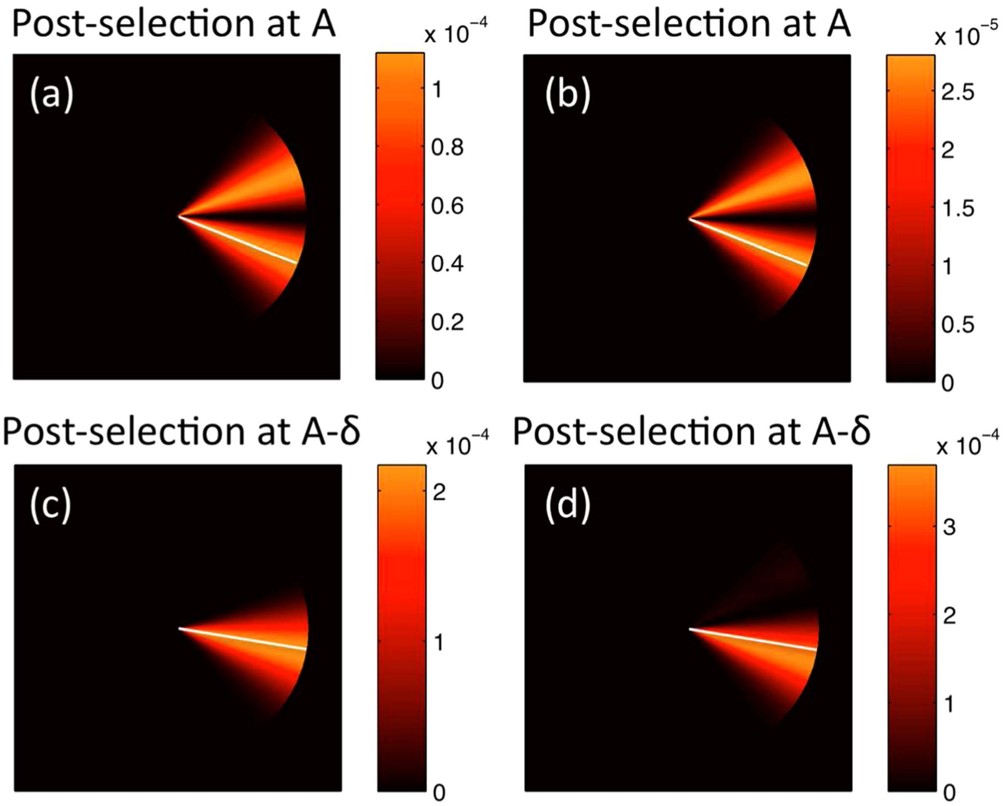

Figure 6. Comparison between conventional interferometry and weak value amplification. The interference fringes in (a) are obtained when the Dove prism induces a rotation of 0.5° between the two angular modes; a similar situation is plotted in (b) for a rotation of 1°. The white lines in the upper panels are at the same angular inclination and are simply a reference to compare the intensity distributions to. With reference to these white lines, it is clear that the difference between (a) and (b) is minimal. Consequently, it is difficult by eye to discern a 1° rotation from 0.5° rotation, or, in fact, whether any rotation has occurred at all. However, this small difference can be easily identified when weak value amplification is performed. Again referencing to the white lines in the lower panels, the difference between the beams in (c) and (d) is evident.

figure 6(c), in which the Dove prism induces a rotation of 0.5° from that shown in figure 6(d) for which the angle is set to 1°.

5. Conclusion

We have described a straightforward scenario in which the WVA technique can outperform conventional measurement. We note that the example that we have provided does not necessarily contradict the existing body of theoretical work that claims WVA to be sub-optimal relative to more conventional measurement strategies, for ideal or near-ideal experiments. Rather, we have concerned ourselves with the case of detection using the naked eye, which possesses a large dynamic range that can easily accommodate intensity losses due to post-selection, provided that a sufficient number of photons are initially available for the measurement. We conclude that, like most experimental protocols, WVA is a procedure from which a benefit can be derived under many—but by no means all—experimental circumstances. More broadly, WVA is just one example of how weak measurement can be used to measure quantities that are sometimes inaccessible to experimentalists [30].

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